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Exact results for the emission from one and two atoms in an ideal cavity at multiphoton resonance

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Abstract. We examine the emission from the systems of one or two two-level atoms in an ideal cavity with one mode at multiphoton resonance. Exact results for the two-time dipole correlation function and the time-dependent spectra of multiphoton-induced fluorescence are presented.

1. Introduction

The Jaynes-Cummings model of a two-level atom interacting with a single mode of the electromagnetic field in a lossless cavity is one of the few exactly soluble models in quantum optics. For many years, theorists have analysed various aspects of this model (for reviews see Yoo and Eberly (1985), Haroche and Raimond (1985)). A number of interesting effects such as vacuum-field Rabi oscillations, collapse and revival have been predicted and then observed experimentally (Kaluzny *et al* 1983, Meschede *et al* 1985, Gallas *et al* 1985, Rempe *et al* 1987). Recently, Sanchez-Mondragon *et al* (1983) and Agarwal (1985) have calculated the emission spectrum for fluorescence photons in this model. The fluorescence and absorption spectra have also been obtained for the model of a single-atom one-mode system with cavity damping (Agarwal and Puri 1986, Bullough *et al* 1986) and for the two-atom generalised models (Agarwal 1985, Cheltsov 1986). It has been shown that the emission of an atom in a cavity is drastically modified compared with its behaviour in free space. For example, Sanchez-Mondragon *et al* (1983) have shown that the spontaneous emission spectrum of an atom excited in a lossless cavity has a doublet structure arising from vacuum-field Rabi oscillations. The cooperative and multiphoton-transition effects (Agarwal 1985, Ho *et al* 1986) make the structure of the spectrum very complicated and thus might lead to difficulties in the study of such a spectrum.

In this paper we examine the emission from one and two two-level atoms interacting through the multiphoton-transition mechanism with one resonant mode in an ideal cavity. Exact quantum electrodynamic results for the dipole correlation functions and the spectra of multiphoton-induced fluorescence are presented.

The paper is organised as follows. In § 2 we study the single-atom system. Section 3 is devoted to the two-atom system and in § 4 we give a summary.

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2. A single atom

In the rotating wave approximation, the effective Hamiltonian for a single two-level atom interacting with a one-mode radiation field in a lossless resonant cavity through the multiphoton-transition mechanism (Singh 1982, Buck and Sukumar 1984) is

$$H = H_0 + H_{AF} = (\hbar\omega_0 R^z + \hbar\omega_f a^+ a) + \hbar g(a^{+m} R^- + a^m R^+). \tag{1}$$

Here $R^\pm$ and R^z are the spin-$\frac{1}{2}$ operators of the atom, a and a^+ are the usual photon operators of the quantised cavity mode, ω_f and ω_0 are the frequencies of the cavity mode and the atom, m is the photon multiple and g is the coupling constant. The exact multiphoton resonance $\omega_0 = m\omega_f$ is assumed to occur.

We assume that the atom was initially in the excited state $|+\rangle$ and the cavity field was in an arbitrary state $\rho_F = \sum_{nn'} p_{nn'}|n\rangle\langle n'|$, i.e. the initial density matrix is

$$\rho(0) = (|+\rangle\langle +|) \otimes \rho_F = \sum_{nn'} p_{nn'}|+; n\rangle\langle +; n'|. \tag{2}$$

By solving the corresponding equations of motion the wavefunctions $\exp(-iH_{AF}t/\hbar)|\pm; n\rangle$ in the interaction picture are found to be

$$\begin{aligned} \exp(-iH_{AF}t/\hbar)|+; n\rangle &= \cos(\Omega_{n+m}t)|+; n\rangle - i\sin(\Omega_{n+m}t)|-; n+m\rangle \\ \exp(-iH_{AF}t/\hbar)|-; n\rangle &= \cos(\Omega_n t)|-; n\rangle - i\sin(\Omega_n t)|+; n-m\rangle \end{aligned} \tag{3}$$

where

$$\Omega_n = g[n(n-1)\ldots(n-m+1)]^{1/2} = g[n!/(n-m)!]^{1/2}. \tag{4}$$

Then, from the definition of the Heisenberg operators we obtain for the two-time dipole correlation function $D(t, \tau)$ the expression

$$D(t, \tau) \equiv \langle R^+(t+\tau)R^-(t)\rangle = \sum_n p_{nn} D_n(t, \tau) \tag{5}$$

where

$$\begin{aligned} D_n(t, \tau) = \tfrac{1}{4} e^{i\omega_0\tau}\{&\cos[2\Omega_{n+m}t + (\Omega_{n+m} + \Omega_n)\tau] + \cos[2\Omega_{n+m}t + (\Omega_{n+m} - \Omega_n)\tau] \\ &+ \cos[(\Omega_{n+m} + \Omega_n)\tau] + \cos[(\Omega_{n+m} - \Omega_n)\tau]\}. \end{aligned} \tag{6}$$

We proceed to the study of the emission spectrum of the atom. No source of relaxation has been assumed in the model. Therefore, the only source of linewidth is the width of the detector. Following Eberly and Wodkiewicz (1977) we define the time-dependent spectrum of the multiphoton-induced fluorescence radiated into other modes (Agarwal and Puri 1986, Sanchez-Mondragon *et al* 1983) as

$$S(\nu, T) = \Gamma\beta \int_0^T dt_1 \int_0^T dt_2 \langle R^+(t_1)R^-(t_2)\rangle \exp[-(\Gamma - i\nu)(T - t_1) - (\Gamma + i\nu)(T - t_2)] \tag{7}$$

where Γ is the bandwidth of the detecting mechanism, T is the time at which the spectrum is evaluated, and β is a measure of fluorescence into other modes. Expression (7) can be transformed so that we have only to evaluate the time-ordered correlation function $D(t, \tau)$, $\tau > 0$. Then, taking into account (5) and (6), one can show that

$$S(\nu, T) = \sum_n p_{nn} S_n(\nu, T) \tag{8}$$

where

$$S_n(\nu, T) = 2\Gamma\beta \operatorname{Re} \int_0^T d\tau \exp[(\Gamma - i\nu)\tau] \int_0^{T-\tau} dt \exp[-2\Gamma(T-t)] D_n(t, \tau)$$

$$= \tfrac{1}{4}\Gamma\beta \operatorname{Re} \sum_{K,j} (\eta_j + 2i\Gamma)^{-1} ((\nu - \omega_0 + \lambda_k - \eta_j - i\Gamma)^{-1}$$

$$\times \{\exp(-i\eta_j T) - \exp[-i(\nu - \omega_0 + \lambda_k - i\Gamma)T]\}$$

$$+ (\nu - \omega_0 + \lambda_k + i\Gamma)^{-1} \{\exp[-i(\nu - \omega_0 + \lambda_k - i\Gamma)T] - \exp(-2\Gamma T)\})$$

$$+ (\nu - \omega_0) \to -(\nu - \omega_0) \tag{9}$$

and

$$\eta_1 = 0 \qquad \eta_2 = 2\Omega_{n+m}$$

$$\lambda_1 = \Omega_{n+m} - \Omega_n \qquad \lambda_2 = \Omega_{n+m} + \Omega_n. \tag{10}$$

If we consider the long-time limit of (9): $\Gamma T \gg 1$, and if we ignore the small oscillating terms (Agarwal and Puri 1986) corresponding to $\exp(-i\eta_2 T)$, then we obtain

$$S_n(\nu, T \to \infty) = \frac{\Gamma\beta/8}{\Gamma^2 + (\nu - \omega_0 + \Omega_{n+m} - \Omega_n)^2}$$

$$+ \frac{\Gamma\beta/8}{\Gamma^2 + (\nu - \omega_0 + \Omega_{n+m} + \Omega_m)^2} + (\nu - \omega_0) \to -(\nu - \omega_0). \tag{11}$$

It is seen from (8)-(10) that if initially the field is in a Fock state $|n\rangle$, then the fluorescence spectrum $S(\nu, T) = S_n(\nu, T)$ consists of several lines whose positions and widths are determined by $\omega_0 \pm (\Omega_{n+m} \pm \Omega_n)$ and Γ. Below, we consider two cases of general interest in the narrow-band detection limit.

Case (1): $n \leqslant m - 1$. It is seen from (4) that $\Omega_n = 0$ in this case. The spectrum $S_n(\nu, T)$ then exhibits a doublet with peaks at $\nu = \omega_0 \pm \Omega_{n+m}$ if $\Gamma \ll \Omega_{n+m}$. Note that the particular case $n = 0$ corresponds to spontaneous emission and hence our result is in agreement with the result of Sanchez-Mondragon *et al* (1983) and Agarwal (1985) showing the doublet structure of the spontaneous emission spectrum in the situation with one-photon-resonance $m = 1$.

In the general situation with multiphoton resonance the separation of the two peaks at $\nu = \omega_0 \pm g\sqrt{m!}$ of the spontaneous emission spectrum is just equal to the frequency $2g\sqrt{m!}$ of the vacuum-field Rabi oscillations and is called the multiphoton-resonance vacuum-field Rabi splitting (Agarwal 1985). This splitting is reminiscent of the fluorescence line splitting predicted in the presence of an intense driving laser field by Mollow (1969) and observed experimentally (Schuda *et al* 1974, Hartig *et al* 1976, Grove *et al* 1977) but it is not the same because it occurs here in the absence of photons in the cavity mode.

Case (2): $n \gg m^2$. From the asymptotic expression

$$\Omega_{n+m} - \Omega_n \approx \tfrac{1}{2} g m^2 n^{m/2-1}$$

$$\Omega_{n+m} + \Omega_n \approx 2g(n^{m/2} + \tfrac{1}{4} m n^{m/2-1}) \qquad n \gg m^2 \tag{12}$$

we find that the spectrum $S_n(\nu, T)$ with $n \gg m^2$ has:

case (2a), four peaks at

$$\nu = \omega_0 \pm \tfrac{1}{2} g m^2 n^{m/2-1} \qquad \nu = \omega_0 \pm 2g(n^{m/2} + \tfrac{1}{4} m n^{m/2-1}) \tag{13}$$

if $\Gamma \ll gm^2 n^{m/2-1}$, and

case (2b), three peaks at

$$\nu = \omega_0 \qquad \nu = \omega_0 \pm 2g\sqrt{n} \tag{14}$$

if $m = 1$ and $\sqrt{n} \gg g/\Gamma, \Gamma/g$ (Agarwal 1985). If we consider the long-time limit, then we find from (11) that the four peaks in case (2a) are equally intense, whereas the central peak in case (2b) is twice as intense as the side peaks. The triplet obtained in case (2b) is in fact the exact analogue of the Mollow triplet of intense-laser line splitting (Mollow 1969, Eberly *et al* 1980).

3. Two-atom system

In this section, we examine the effect of cooperativity on the fluorescence spectrum. In particular, we consider what happens to the spectrum owing to the presence of two excited atoms with multiphoton transitions in a lossless resonant cavity. The two-atom generalisation of the Hamiltonian (1) is given by

$$H = H_0 + H_{AF}$$

$$H_0 = \hbar\omega_f a^+ a + \sum_{j=1}^{2} \hbar\omega_0 R_j^z \equiv \hbar\omega_f a^+ a + \hbar\omega_0 R^z \tag{15}$$

$$H_{AF} = \sum_{j=1}^{2} \hbar g(a^{+m} R_j^- + a^m R_j^+) \equiv \hbar g(a^{+m} R^- + a^m R^+)$$

where $R^{z,+,-} = \sum_{j=1}^{2} R_j^{z,+,-}$ are now spin-1 collective operators. We calculate the correlation function $D(t, \tau)$ and the spectrum $S(\nu, T)$ assuming that each atom is initially in the excited state and that the field is in an arbitrary state $\rho_F = \sum_{nn'} p_{nn'} |n\rangle\langle n'|$. Let us denote the atomic spin-1 eigenstates $|R = 1, M_R = 0, \pm 1\rangle$ by $|S_{0,\pm 1}\rangle$, i.e.

$$|S_0\rangle = (|+,-\rangle + |-,+\rangle)/\sqrt{2} \qquad |S_1\rangle = |+,+\rangle \qquad |S_{-1}\rangle = |-,-\rangle. \tag{16}$$

It follows from the structure of the Hamiltonian (15) that the time-dependent wavefunctions $\exp(-iHt/\hbar)|S_1; n\rangle$, $\exp(-iHt/\hbar)|S_0; n+m\rangle$ and $\exp(-iHt/\hbar)|S_{-1}; n+2m\rangle$ are linear superpositions of the three basic states $|S_1; n\rangle$, $|S_0; n+m\rangle$ and $|S_{-1}; n+2m\rangle$. The coefficients that appear in these superpositions are to be obtained from the solutions of the Schrödinger equations. The dipole correlation function $D(t, \tau) = \langle R^+(t+\tau) R^-(t)\rangle$ is calculated in terms of the above coefficients to be

$$D(t, \tau) = \sum_n p_{nn} D_n(t, \tau) \tag{17a}$$

$$D_n(t, \tau) = 2\, e^{i\omega_0 \tau}[\alpha_n(t+\tau)(\alpha_n(t)\mu_n(\tau) - \beta_n(t)\chi_n(\tau)) + \beta_n(t+\tau)(\alpha_n(t)\chi_n(\tau) + \beta_n(t)\varphi_n(\tau))]. \tag{17b}$$

Here the notation

$$\alpha_n(t) = 1 - \frac{g^2 q_{n+m}}{\tilde{\Omega}_{n+m}^2} \sin^2[\tilde{\Omega}_{n+m} t]$$

$$\beta_n(t) = \frac{g(q_{n+m}/2)^{1/2}}{\tilde{\Omega}_{n+m}} \sin[2\tilde{\Omega}_{n+m} t]$$

$$\chi_n(t) = \frac{g(q_{n+m}/2)^{1/2}}{\tilde{\Omega}_n} \sin[2\tilde{\Omega}_n t] \tag{18}$$

$$\mu_n(t) = \cos[2\tilde{\Omega}_n t]$$

$$\varphi_n(t) = 1 - \frac{g^2 q_{n+m}}{\tilde{\Omega}_n^2} \sin^2[\tilde{\Omega}_n t]$$

and

$$q_n = n!/(n-m)! \qquad \tilde{\Omega}_n = g((q_n + q_{n+m})/2)^{1/2} \tag{19}$$

has been introduced. Note that $D_n(t, \tau)$ has the structure

$$D_n(t, \tau) = 2\,\mathrm{e}^{\mathrm{i}\omega_0\tau} \sum_{kj} A_{kj} \cos[\tilde{\lambda}_k \tau + \tilde{\eta}_j t]. \tag{20}$$

Therefore, the time-dependent fluorescence spectrum defined by (7) has the form (Agarwal and Puri 1986):

$$S(\nu, T) = \sum_n p_{nn} S_n(\nu, T) \tag{21a}$$

$$\begin{aligned} S_n(\nu, T) = 2\Gamma\beta\ \mathrm{Re} \sum_{kj} A_{kj} (\tilde{\eta}_j + 2\mathrm{i}\Gamma)^{-1} \\ \times ((\nu - \omega_0 + \tilde{\lambda}_k - \tilde{\eta}_j - \mathrm{i}\Gamma)^{-1} \{\mathrm{e}^{-\mathrm{i}\tilde{\eta}_j T} - \exp[-\mathrm{i}(\nu - \omega_0 + \tilde{\lambda}_k - \mathrm{i}\Gamma) T]\} \\ + (\nu - \omega_0 + \tilde{\lambda}_k + \mathrm{i}\Gamma)^{-1} \{\exp[-\mathrm{i}(\nu - \omega_0 + \tilde{\lambda}_k - \mathrm{i}\Gamma) T] - \exp(-2\Gamma T)\}) \\ + (\nu - \omega_0) \to -(\nu - \omega_0). \end{aligned} \tag{21b}$$

Here $S_n(\nu, T)$ is the collective fluorescence spectrum of the two-atom system in the situation when the cavity field is initially in the Fock state $|n\rangle$. Using (17*b*), (18), (20) and (21*b*) one can show that in the long-time limit the spectrum $S_n(\nu, T \to \infty)$ is explicitly given by

$$\begin{aligned} S_n(\nu, T \to \infty) = \frac{g^4 q_{n+2m}^2}{4\tilde{\Omega}_{n+m}^4} \frac{\Gamma\beta}{\Gamma^2 + (\nu - \omega_0 - 2\tilde{\Omega}_n)^2} + \frac{g^4 q_n q_{n+m}}{8\tilde{\Omega}_n^2 \tilde{\Omega}_{n+m}^2} \frac{\Gamma\beta}{\Gamma^2 + (\nu - \omega_0 - 2\tilde{\Omega}_{n+m})^2} \\ + \frac{g^4 q_{n+m}^2}{16\tilde{\Omega}_{n+m}^2} \left(\frac{1}{\tilde{\Omega}_n} - \frac{1}{\tilde{\Omega}_{n+m}}\right)^2 \frac{\Gamma\beta}{\Gamma^2 + (\nu - \omega_0 - 2\tilde{\Omega}_{n+m} - 2\tilde{\Omega}_n)^2} \\ + \frac{g^4 q_{n+m}^2}{16\tilde{\Omega}_{n+m}^2} \left(\frac{1}{\tilde{\Omega}_n} + \frac{1}{\tilde{\Omega}_{n+m}}\right)^2 \frac{\Gamma\beta}{\Gamma^2 + (\nu - \omega_0 - 2\tilde{\Omega}_{n+m} + 2\tilde{\Omega}_n)^2} \\ + (\nu - \omega_0) \to -(\nu - \omega_0). \end{aligned} \tag{22}$$

This spectrum will have as many as from one to eight splittings depending on the resolution of the following frequencies:

$$\begin{aligned} \nu_{\pm 1} &= \omega_0 \pm 2(\tilde{\Omega}_{n+m} - \tilde{\Omega}_n) \\ \nu_{\pm 2} &= \omega_0 \pm 2\tilde{\Omega}_n \\ \nu_{\pm 3} &= \omega_0 \pm 2\tilde{\Omega}_{n+m} \\ \nu_{\pm 4} &= \omega_0 \pm 2(\tilde{\Omega}_{n+m} + \tilde{\Omega}_n). \end{aligned} \tag{23}$$

The linewidths are defined by the width Γ of the detector. Note that the expressions of $\nu_{\pm 4}$ in (23) describe the quantum electrodynamic analogue of the so-called cooperative additional sidebands (Senitzky 1978, Agarwal *et al* 1980, Mavroyannis 1980, Cordes 1982, Ficek *et al* 1983). The existence of these higher harmonics in the vacuum and weak cavity fields has recently been shown by Agarwal (1985) and Cheltsov (1986). We now consider in detail two cases of general interest in the narrow-band detection limit.

Case (1): $n \leqslant m-1$. In this case $q_n = 0$ and therefore the second term on the right-hand side of (22) is also equal to zero. The spectrum (22) then has six peaks at $\nu = \omega_0 \pm 2\tilde{\Omega}_n$ and $\nu = \omega_0 \pm 2(\tilde{\Omega}_{n+m} \pm \tilde{\Omega}_n)$ if $\Gamma \ll 2(\tilde{\Omega}_{n+m} - \tilde{\Omega}_n)$. In particular, we find that the spontaneous emission spectrum $S_{n=0}(\nu, T \to \infty)$ peaks at the frequencies

$$\nu_{\pm 1} = \omega_0 \pm g\{\sqrt{2}[m! + (2m)!/m!]^{1/2} - [2(m!)]^{1/2}\} \tag{24a}$$

$$\nu_{\pm 2} = \omega_0 \pm g[2(m!)]^{1/2} \tag{24b}$$

$$\nu_{\pm 4} = \omega_0 \pm g\{\sqrt{2}[m! + (2m)!/m!]^{1/2} + [2(m!)]^{1/2}\}. \tag{24c}$$

The relations between the heights of the peaks (24*a*), (24*b*) and (24*c*) are

$$I_{\pm 1} : I_{\pm 2} : I_{\pm 4} = \left[\left(m! + \frac{(2m)!}{m!}\right)^{1/2} + (m!)^{1/2}\right]^2 : \frac{4}{m!}\left(\frac{(2m)!}{m!}\right)^2 : \left[\left(m! + \frac{(2m)!}{m!}\right)^{1/2} - (m!)^{1/2}\right]^2. \tag{25}$$

When $m = 1$ these expressions reduce to those obtained by Agarwal (1985) for the vacuum-field Rabi splittings of the spectrum of a two-atom system with one-photon resonance.

Case (2): $n \gg m^2$. From (19) we obtain the following asymptotic expressions:

$$\begin{aligned} &q_n \approx q_{n+m} \approx q_{n+2m} \approx n^m \\ &\tilde{\Omega}_n \approx g(n^{m/2} + \tfrac{1}{4}mn^{m/2-1}) \\ &\tilde{\Omega}_{n+m} \approx g[n^{m/2} + \tfrac{1}{4}m(2m+1)n^{m/2-1}] \end{aligned} \tag{26}$$

for $n \gg m^2$. Therefore, the spectrum $S_n(\nu, T \to \infty)$ with $n \gg m^2$ will have eight peaks at the frequencies

$$\begin{aligned} &\nu_{\pm 1} \approx \omega_0 \pm gm^2 n^{m/2-1} \\ &\nu_{\pm 2} \approx \omega_0 \pm 2g(n^{m/2} + \tfrac{1}{4}mn^{m/2-1}) \\ &\nu_{\pm 3} \approx \omega_0 \pm 2g[n^{m/2} + \tfrac{1}{4}m(2m+1)n^{m/2-1}] \\ &\nu_{\pm 4} \approx \omega_0 \pm 4g[n^{m/2} + \tfrac{1}{4}m(m+1)n^{m/2-1}] \end{aligned} \tag{27}$$

if

$$\Gamma \ll gm^2 n^{m/2-1}. \tag{28}$$

The relations between the heights of these peaks are approximately found from (22) and (26) to be

$$I_{\pm 1} : I_{\pm 2} : I_{\pm 3} : I_{\pm 4} \approx 2 : 2 : 1 : (m^4/8n^2). \tag{29}$$

Since $m^4/8n^2 \ll 1$, the extreme side peaks $\nu_{\pm 4}$ are very weak compared with the other peaks. For the case $\Gamma \sim gm^2 n^{m/2-1}$, it is possible that instead of the two peaks $\nu_{\pm 1}$ one has the new central peak $\nu_0 = \omega_0$. The number of peaks then is seven. Increasing the detection bandwidth Γ, we may decrease the number of spectrum peaks up to 1. It should be noted here that the narrow-band detection condition (28) implies, in the case $m \geqslant 3$, the requirement of a strong cavity field, but in the case $m = 1$ it restricts the field intensity. In the case of one-photon resonance and strong cavity field: $m = 1, \sqrt{n} \gg g/\Gamma, \Gamma/g$, the spectrum $S_n(\nu, T \to \infty)$ has only five peaks at the frequencies

$$\nu_0 = \omega_0 \qquad \nu_{\pm 2} = \omega_0 \pm 2g\sqrt{n} \qquad \nu_{\pm 4} = \omega_0 \pm 4g\sqrt{n} \tag{30}$$

since $\nu_{\pm 1} \to \nu_0$, $\nu_{\pm 3} \to \nu_{\pm 2}$. The relations between the heights of these peaks now become

$$I_0 : I_{\pm 2} : I_{\pm 4} = 4 : 3 : 1/8n^2. \tag{31}$$

If n is very large so that the extreme sidebands $\nu_{\pm 4} = \omega_0 \pm 4g\sqrt{n}$ can be neglected, then the spectrum $S_n(\nu, T \to \infty)$ will have a three-peak structure. The results expressed by (30) and (31), and in particular, the decrease of the heights of the cooperative sidebands $\nu_{\pm 4}$ as $1/n^2$ as $n \to \infty$ are in agreement with the results of Cheltsov (1986) but are obtained having applied a different method.

Finally, we emphasise that for an arbitrary state of the cavity field characterised by the photon distribution p_{nn}, one has to average the spectra $S_n(\nu, T)$ in (9) and (21*b*) with respect to the distribution p_{nn}. All the spectral characteristics will essentially depend on the statistical properties of p_{nn}. In particular, the linewidths, and consequently the number of peaks, will be determined not only by Γ but also by the dispersion of this distribution.

4. Summary

We have investigated the emission from one and two two-level atoms in an ideal cavity with one mode at multiphoton resonance. The exact quantum electrodynamic results for the two-time dipole correlation functions and the time-dependent spectra of multiphoton-induced fluorescence have been obtained. We have examined the general situation with an arbitrary state of the quantised cavity mode. The multiphoton-resonance vacuum-field Rabi splittings have been calculated. The structure of the spectra in the situation with a Fock state of the cavity field has also been studied. The side peaks at the higher harmonics $\nu_{\pm 4}$ of the collective spectrum $S_n(\nu, T \to \infty)$ have been shown to exist in vacuum and weak cavity fields, their intensities tending to zero as m^4/n^2 as $n \to \infty$. Our results clearly show the complications to the spectra caused by the quantum discrete nature of the cavity field, the photon multiplicity of resonance and the atomic cooperativity.

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